

# Discovering Spatial High Utility Itemsets in Spatiotemporal Databases

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## ABSTRACT

In real-world databases, high utility itemset (HUI) is an important class of regularities. Most previous studies have focused on mining HUIs in transactional databases and did not consider the spatiotemporal characteristics of items. In this study, a more flexible model of spatial HUIs (SHUIs) that exist in spatiotemporal databases is proposed. In a spatiotemporal database (*STD*), an itemset is said to be an SHUI if its *utility* is not less than a user-specified *minimum utility* and the distance between any two of its items is not more than a user-specified *maximum distance*. Identifying SHUIs is very challenging because the generated itemsets do not satisfy the anti-monotonic property. In this study, we present two novel pruning techniques for reducing computational costs. Moreover, a fast single scan algorithm is presented for effectively evaluating all SHUIs in a *STD*. Furthermore, two case studies are presented, in which the proposed model is used to identify useful information in traffic congestion data and air pollution data.

## CCS CONCEPTS

• Information systems → Association rules.

## KEYWORDS

Data mining, utility itemset mining, pattern mining and spatiotemporal databases

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## 1 INTRODUCTION

In data mining, frequent itemset mining (FIM) [1] is an important model, and its mining algorithms can be used to discover all itemsets in a transactional database that satisfies a user-specified *minimum support* (*minSup*) constraint. The latter indicates that the minimum number of transactions that an itemset must cover in the data. Because the *support* measure is only used to determine the interestingness of an itemset, the model implicitly assumes that all items within the data have uniform weight and that an item at most appears only once in a transaction. Both these assumptions limit the applicability of FIM because the items in many real-world databases have non-uniform weights and can occur multiple times in a transaction. For example, in a supermarket, items have different prices and a customer can purchase multiple quantities of an item such as three bottles of milk, six bread rolls, and a bottle of wine. Because FIM algorithms disregard the purchase quantities of items and their unit prices, they may encounter many uninteresting frequent itemsets. Therefore, the generation of revenue would be low and many rare itemsets that could result in the generation of high revenue would be missed. This problem is referred to as the "rare item problem" [12].

To address the rare item problem, Yao et al. [25] proposed high utility itemset mining (HUIM), which extends considering the items' internal utility (such as the number of occurrences of an item within a transaction) and external utility (such as the weight of an item). Given a (non-binary) transactional database, HUIM aims to find all high utility itemsets (HUIs), i.e., itemsets having a *utility* that is not less than a user-specified *minimum utility* (*minUtil*) constraint. The *utility* of an itemset is the sum of its utilities in all transactions. HUIM has a number of applications such as market-basket analysis [24], mobile commerce [23], click stream analysis [2], biomedicine [16], and cross-marketing [9]. Moreover, mining HUIs have inspired several other important data mining tasks such as high utility sequential pattern mining [26] and high utility periodic pattern mining [8].

Although HUIM and its variants consider an item's weight and its occurrence *frequency* in a transaction, they disregard spatiotemporal characteristics of the item. Consequently, they fail to discover only the HUIs that have items close to one another in a spatiotemporal database. To address this issue, we introduced spatial HUI

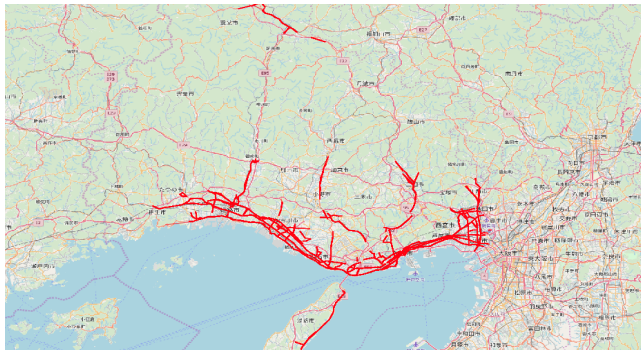


Figure 1: Spatial visualization of roads in Kobe, Japan

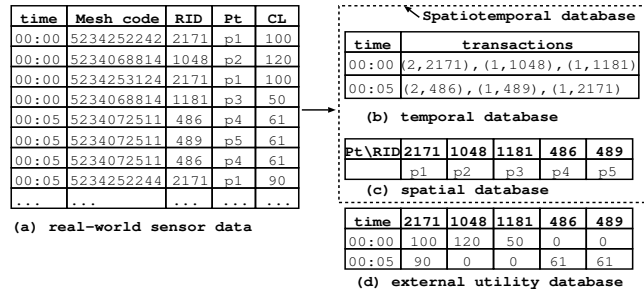


Figure 2: Representing real-world sensor data as a spatiotemporal database and an external utility database. The terms "Pt," "RID," and "CL" represent the point, road identifier, and congestion length in meters, respectively.

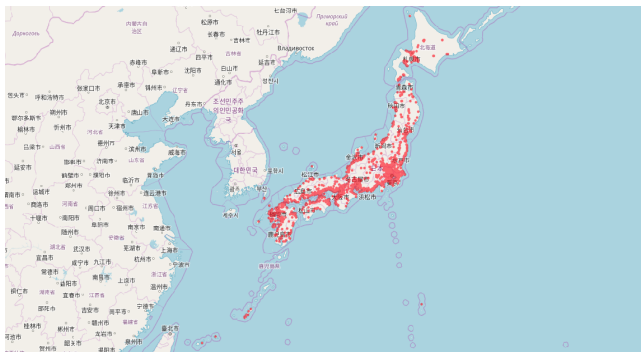


Figure 3: Spatial visualization of AEROS sensors

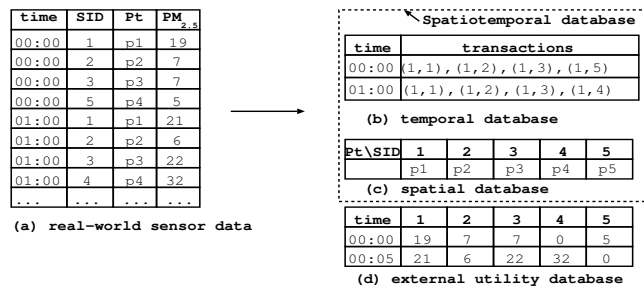


Figure 4: Representing the real-world sensor data as a spatiotemporal database and an external utility database. The terms "Pt" and "SID" represent the point and sensor identifier, respectively. PM<sub>2.5</sub> is measured in micro-gram per cubic-meter (i.e.,  $\mu\text{g}\cdot\text{m}^3$ )

mining (SHUIM). Herein we present two business applications of SHUIM.

The first application of SHUIM is improving the road traffic safety in smart cities. To measure and monitor congestion at various road segments, Japan Road Traffic Information Center (JARTIC) has set up a sensor network. Figure 1 shows the road network covered by congestion measuring sensors in Kobe, Japan. The data generated by these sensors represent a spatiotemporal database and an external utility database. Figure 2(a) shows a portion of the actual data generated by the sensors. Figures 2(b), (c), and (d) show the temporal database, spatial database, and external utility database generated from Figure 2(a), respectively. An SHUI generated from these databases, such as a total of 5 km of congestion, was observed on the set of roads f2171; 1181g and provided information regarding the set of neighboring roads where a significant length of congestion was observed. At the time of disasters, the above mentioned information can be very useful to users for various purposes such as diverting the traffic, alerting the pedestrians, and suggesting a patrol path for the police. Japanese Industrial Standard X 0410 regional mesh code (or simply, mesh code) [4] represents a portion of the square grid on the Earth's surface. Because a road segment can pass through multiple mesh codes, the congestion length observed by a sensor is equally split among its mesh codes. Thus, the internal

utility of an item in the temporal database represents the number of occurrences of a road segment with various mesh codes at a particular timestamp. The terms "p1" to "p5" in Figure 2 represent the points (i.e., latitude and longitude) of the corresponding road segments.

The second application of SHUIM is improving public health by identifying highly polluted geographical regions. Air pollution is a major cause of cardio-respiratory problems for people living in Japan. Thus, to tackle pollution, the Japanese Ministry of Environment has set up the Atmospheric Environmental Regional Observation System (AEROS). This system comprises air pollution measuring sensors located all over Japan (see Figure 3). The data generated by these sensors is a spatiotemporal database and an external utility database, as shown in Figure 4. An SHUI identified in this pollution database indicates that a total of  $1250\mu\text{g}\cdot\text{m}^3$  of PM<sub>2.5</sub><sup>1</sup> was observed by a set of sensors f1; 4; 5g that are close to one other. This provides information regarding neighboring sensors (or geographical regions) where large quantities of air pollution have been observed. This information is useful to users of pollution control board and helps them take appropriate steps such as increasing the vegetation, alerting the people to remain indoors

<sup>1</sup>PM<sub>2.5</sub> refers to atmospheric particulate matter that have a diameter of <2.5 micrometers.

at certain times of the day, and proposing new policies to regulate industrial emissions.

HUIM algorithms cannot be directly employed for identifying SHUIs in spatiotemporal databases because of the following reasons:

HUIM algorithms implicitly assume that transactions occur at a fixed time interval and the external utility of an item remains unchanged in the entire data. However, a SHUIM algorithm has to consider that transactions within the data can occur at irregular time intervals and the external utilities of items (such as congestion length on a road segment and  $PM_{2.5}$  value recorded by a sensor) can vary over time.

HUIM algorithms determine the interestingness of an itemset without considering the spatial information of its items. Alternatively, a SHUIM algorithm has to determine the interestingness of an itemset by considering both its *utility* and the closeness (or proximity) of its items.

Identifying SHUIs in spatiotemporal databases is a computationally expensive process because the *utility* function does not satisfy the *convertible anti-monotone*, *convertible monotone*, or *convertible succinct properties* [19]. The pruning techniques of existing HUIM algorithms can be employed to reduce computational costs; however, such techniques are inefficient because they disregard the spatial information of items.

With this motivation, we propose a more flexible model of SHUIs in spatiotemporal databases. In a spatiotemporal database (*STD*), an itemset is said to be an SHUI if its *utility* is not less than a user-specified *minimum utility* and the distance between any two of its items is not more than a user-specified *maximum distance*. To calculate an upper bound on the *utility* of itemsets, two new measures, *Probable Maximum Utility (PMU)* and *Neighborhood Sub-tree Utility (NSU)*, are proposed. Moreover, to effectively reduce the search space, two novel pruning techniques employing the *PMU* and *NSU* measures have been proposed. Furthermore, a fast single scan algorithm, called SHUI-Miner is presented to effectively determine all SHUIs in a spatiotemporal database. The proposed SHUI-Miner algorithm is based on the renowned efficient high-utility itemset mining (EFIM) algorithm [27]. Experimental results show that SHUI-Miner is efficient. We also present two case studies in which we applied the proposed model to identify useful information in traffic congestion and air pollution data.

The remainder of this study is organized as follows. In Section 2, we discuss the literature on spatial frequent itemset mining and HUI mining. In Section 3, we introduce the proposed model of SHUI. In Section 4, we introduce the SHUI-Miner algorithm. In Section 5, we present the experimental results. Finally, in Section 6, we provide conclusion of this study and discuss future research directions.

## 2 RELATED WORK

### 2.1 Spatial co-occurrence itemset mining

Since the introduction of frequent itemsets in [1], the problem of identifying spatiotemporal co-occurrence itemsets (or association rules) in spatiotemporal databases has received considerable attention of researchers [6, 7, 17, 22]. These algorithms can be broadly classified into distance-based approaches [6, 7] and

transaction-based approaches [17, 22]. Typically, a distance-based approach uses a parameter called prevalence to determine how interesting spatiotemporal co-occurrences occur in the data. Initially, a transaction-based approach clusters data over space and time (i.e., spatial clustering) and then applies traditional association rule mining algorithms to cluster and determine useful associations. Unfortunately, all spatiotemporal co-occurrence itemset mining algorithms determine the interestingness of an itemset by considering only its *support* and disregarding the internal and external utility values of an item. Consequently, most of these algorithms cannot handle numeric data. Nevertheless, the proposed SHUIM model considers the internal and external utility values of items and handles numeric data.

### 2.2 HUI mining

To address the rare item problem in FIM, Cai et al. [5] proposed weighted frequent itemset mining (WFIM). WFIM algorithms [5, 10] can be used to identify all interesting itemsets in a binary transactional database using the weight (or importance) of items. A major limitation of WFIM algorithms is that they do not consider the number of occurrences of an item in a transaction. Yao et al. [25] proposed HUIM, which considers the importance of items and their occurrence *frequency* in every transaction. To circumvent the fact that *utility* is not anti-monotonic and to identify all HUIs, algorithms such as IHUP [3], PB [11], Two-Phase [15], BAHUI [21], UP-Growth and UP-Growth+ [13, 14], and MU-Growth [28] utilize the *transaction weighted utility*  $TWU^o$  to prune the search space.  $TWU$  is an upper bound on the utility of itemsets. Note that the abovementioned algorithms operate in two phases. In the first phase, they identify candidates of HUIs by calculating their  $TWUs$  to prune the search space. If an itemset has a  $TWU$  greater than  $minUtil$ , then it is considered as a candidate itemset because the itemset or its supersets may be HUIs. Otherwise, if an itemset has a  $TWU$  less than  $minUtil$ , it is discarded. In the second phase, these algorithms scan the database to calculate the exact utility of all candidates to filter those that are low-utility itemsets; unfortunately, all of the abovementioned algorithms suffer from the problem of generating too many candidates.

Recently, to avoid the problem of candidate generation, single phase HUI mining algorithms (such as d2HUP [13], EFIM [27], and HU-FIMi [24]) were developed. These algorithms use upper bounds that are tighter than the  $TWU$  to prune the search space and can immediately obtain the exact utility of any itemset to determine if it should be outputted. Unfortunately, all HUIM algorithms disregard the spatiotemporal characteristics of items within the data. The proposed SHUIM model generalizes HUIM by considering the spatiotemporal characteristics of the items. Thus, the proposed SHUIM model is novel and distinct from HUIM.

## 3 PROPOSED MODEL

Let  $I = \{i_1; i_2; \dots; i_m\}$ ,  $m \geq 1$ , be a set of items. Let  $p^{ij}; ts^o$  denote the **external utility** of an item  $ij \in I$  at a timestamp  $1 \leq ts \leq n$ . Let  $P^{ij^o} = p^{ij}; 1^o \dots [p^{ij}; n^o$  denote the set of all external utility values of  $ij$  in the data. The (external) **utility database**,  $UD$ , refers to the set of external utility values of all items in  $I$ ,

ts	Items
1	<sup>1</sup> a; <sup>2</sup> °; <sup>1</sup> b; <sup>3</sup> °; <sup>1</sup> g; <sup>2</sup> °; <sup>1</sup> f; <sup>1</sup> °
2	<sup>1</sup> a; <sup>1</sup> °; <sup>1</sup> c; <sup>3</sup> °; <sup>1</sup> f; <sup>4</sup> °; <sup>1</sup> g; <sup>1</sup> °
3	<sup>1</sup> d; <sup>3</sup> °; <sup>1</sup> f; <sup>2</sup> °; <sup>1</sup> g; <sup>3</sup> °
4	<sup>1</sup> b; <sup>3</sup> °; <sup>1</sup> c; <sup>4</sup> °; <sup>1</sup> d; <sup>1</sup> °
6	<sup>1</sup> b; <sup>4</sup> °; <sup>1</sup> c; <sup>4</sup> °; <sup>1</sup> d; <sup>1</sup> °; <sup>1</sup> e; <sup>1</sup> °
9	<sup>1</sup> a; <sup>1</sup> °; <sup>1</sup> b; <sup>2</sup> °; <sup>1</sup> c; <sup>2</sup> °; <sup>1</sup> e; <sup>3</sup> °; <sup>1</sup> g; <sup>1</sup> °

Table 1: Temporal database

ts/Item	a	b	c	d	e	f	g
1	100	50	0	0	0	100	200
2	50	0	100	0	0	50	100
3	0	0	0	100	0	100	50
4	0	200	200	100	0	0	0
6	0	150	100	200	50	0	0
9	100	100	50	0	150	0	200

Table 2: External utility database

Items	location
a	<sup>1</sup> 0; <sup>0</sup> °
b	<sup>1</sup> 3; <sup>4</sup> °
c	<sup>1</sup> 3; <sup>4</sup> °
d	<sup>1</sup> 6; <sup>0</sup> °
e	<sup>1</sup> 3; <sup>0</sup> °
f	<sup>1</sup> 9; <sup>0</sup> °
g	<sup>1</sup> 12; <sup>0</sup> °

Table 3: Spatial database

i.e.,  $UD = \bigcup_{ij \in I} P^{1ij^\circ}$ . A temporal database,  $D = \{T_1; T_2; \dots; T_n\}$ ,

where  $T_{ts} \subseteq I$  denotes a transaction. Each transaction  $T_{ts}$  is associated with a timestamp  $ts \in \{1; \dots; n\}$ . Every item  $ij \in T_{ts}$  has a positive number  $q^{1ij; T_{ts}^\circ}$ , called its **internal utility**. Generally, the internal utility of an item represents its *frequency* in a transaction. A spatial database,  $SD = \{lat_{ij}; long_{ij}\}_{ij \in I}$  is a collection

of location points of all items in  $I$ . The terms  $lat_{ij}$  and  $long_{ij}$  denote the latitude and longitude information of an item  $ij$ , respectively. (A spatiotemporal database  $STD$  is a combination of  $D$  and  $SD$ . For brevity, we describe SHUIM using  $UD$ ,  $D$ , and  $SD$ .)

**Example 1.** Let  $I = \{a; b; c; d; e; f; g\}$  be a set of all items (or road identifiers). An irregular temporal database generated from  $I$  is shown in Table 1. Table 2 presents the external utilities (or congestion lengths) of all items at various timestamps. Let the unit for measuring congestion length be m. The external utility of item  $a$  at timestamp 1 is  $p^{1a; 1^\circ} = 100$ , and the internal utility of item  $a$  at timestamp 1 is  $q^{1a; 1^\circ} = 2$ . The spatial database shown in Table 3 provides the location information of all items in Table 1. (For brevity, internal utilities have been presented along with the items in Table 1; however, these values can be treated as a separate database similar to that of the external utility database.)

**Definition 1. (Utility of an item in a transaction)** The utility of an item  $ij$  in a transaction  $T_{ts}$ , denoted as  $u^{1ij; T_{ts}^\circ}$ , representing the product of its internal and external utility values, i.e.,  $u^{1ij; T_{ts}^\circ} = p^{1ij; T_{ts}^\circ} \cdot q^{1ij; T_{ts}^\circ}$ .

**Example 2.** Continuing with the previous example, the *utility* of an item  $a$  (i.e., congestion length at road  $a$ ) in the first transaction is  $u^{1a; T_1^\circ} = p^{1a; T_1^\circ} \cdot q^{1a; T_1^\circ} = 2 \cdot 100 = 200$  m.

**Definition 2. (Utility of an itemset in a transaction)** Let  $X \subseteq I$  be an itemset. An itemset is a  $k$ -itemset if it contains  $k$  items. The utility of an itemset  $X$  in a transaction  $T_{ts}$  is  $u^1X; T_{ts}^\circ = \sum_{ij \in X} u^{1ij; T_{ts}^\circ}$  if  $X \subseteq T_{ts}$ ; otherwise,  $u^1X; T_{ts}^\circ = 0$ .

**Example 3.** The set of items "a" and "b," i.e.,  $\{a; b\}$  (or "ab" in short) is an itemset, which contains two items; therefore, it is a 2-itemset. The utility of itemset  $ab$  (or the total length of congestion observed on the roads  $a$  and  $b$  at the timestamp 1) in the transaction  $T_1$  is  $u^{1ab; T_1^\circ} = u^{1a; T_1^\circ} + u^{1b; T_1^\circ} = 200 + 150 = 350$  m.

**Definition 3. (Utility of an itemset in a database)** The utility of an itemset  $X$  in a database  $D$  is defined as  $u^1X^\circ = \sum_{T_{ts} \in \mathcal{G}^1X^\circ} u^1X; T_{ts}^\circ$ , where  $\mathcal{G}^1X^\circ$  denotes the set of all transactions containing  $X$ .

**Example 4.** In Table 1, the itemset  $ab$  appears in the transactions  $T_1$  and  $T_9$ ; therefore,  $\mathcal{G}^1X^\circ = \{T_1; T_9\}$ . The *utility* of itemset  $ab$  (or the total length of congestion observed on the roads  $a$  and  $b$ ) in the entire database is  $u^{1ab^\circ} = u^{1ab; T_1^\circ} + u^{1ab; T_9^\circ} = 350 + 300 = 650$  m. Similarly, the *utility* of itemset  $cd$  in the entire database is  $u^{1cd^\circ} = u^{1cd; T_4^\circ} + u^{1cd; T_6^\circ} = 900 + 600 = 1;500$  m.

**Definition 4. (High utility itemset)** An itemset  $X$  is said to be a HUI if  $u^1X^\circ \geq \minUtil$ , where  $\minUtil$  represents the user-specified minimum utility value.

**Example 5.** If the user-specified  $\minUtil = 1;500$  m, then  $ab$  is not a HUI because  $u^{1ab^\circ} < \minUtil$ , i.e.,  $ab$  is an uninteresting low utility itemset. However,  $cd$  is a HUI because  $u^{1cd^\circ} \geq \minUtil$ .

**Definition 5. (Spatial high utility itemset)** A HUI  $X$  is said to be a spatial high utility itemset if and only if the distance between its items is not more than the user-specified *maximum distance* ( $\maxDist$ ). Thus, an itemset  $X$  is a SHUI if  $u^1X^\circ \geq \minUtil$  and  $\forall i_a; i_b \in X; a \neq b; Dist^1i_a; i_b^\circ \leq \maxDist$ , where  $Dist^1i_a; i_b^\circ$  is a distance function (e.g. Euclidean distance and Geodesic distance) and  $\maxDist$  denotes a user-specified maximum distance constraint.

**Example 6.** Continuing with the previous example, the HUI  $cd$  is also an SHUI because  $Dist^1c; d^\circ \leq \maxDist$ . This SHUI can be expressed as  $cd$  utility = 1500 m. The set of all SHUIs generated from Table 1 are shown in Table 4. Note that Table 5 lists the neighbors of every item at  $\maxDist = 5$ .

Note that all items in a SHUI must be close to one another (i.e., the distance between any two items in a SHUI must always be less than or equal to the user-specified  $\maxDist$ ). If we relax this constraint, then uninteresting itemsets with items far away from the rest can be generated as SHUIs.

**Example 7.** Let  $p = \{0; 0^\circ\}$ ,  $q = \{4; 0^\circ\}$ ,  $r = \{8; 0^\circ\}$ , and  $s = \{12; 0^\circ\}$  be four items located on a straight line and let  $\maxDist = 4$ . If we relax the constraint that all items in a SHUI need not be close to each other, then we may identify  $pqrs$  as a SHUI because  $dist^1p; q^\circ \leq \maxDist$ ,  $dist^1q; r^\circ \leq \maxDist$  and  $dist^1r; s^\circ \leq \maxDist$ . Clearly, the itemset  $pqrs$  is uninteresting because the items  $r$  and  $s$  are far away from  $p$ .

Itemset	utility
c	1600
b	1550
cd	1500
bd	1500

**Table 4: SHUIs generated from Table 1 at  $minUtil = 1500$  and  $maxDist = 5$**

Item	Neighbours
a	bce
b	ade
c	ade
d	bcef
e	abcd
f	dg
g	f

**Table 5: Neighbors of each item at  $maxDist = 5$**

**Definition 6. (Problem definition)** Given a temporal database  $D$ , spatial database  $SD$ , and a utility database  $UD$ , the problem of SHUIM is to identify all itemsets that have a *utility* not less than the user-specified  $minUtil$  and the distance between its items is not more than the user-specified  $maxDist$ . Interestingly, HUIM is a special case of SHUIM problem when  $maxDist = 1$  (or for a very large value).

In the next section, we present the SHUI-Miner algorithm for efficiently identifying all SHUIs in a spatiotemporal database.

## 4 SHUI-MINER

In this section, we first present the basic idea of SHUI-Miner. Next, we explain the algorithm using the database shown in Table 1.

### 4.1 Basic idea

The space of items in a database gives rise to a subset lattice. When mining SHUIs, the itemset lattice is a conceptualization of the search space. However, reducing this search space is a challenging task because the generated SHUIs do not satisfy the *convertible anti-monotonic*, *convertible monotonic*, or *convertible succinct properties* [18]. To address this challenge, SHUI-Miner employs two new *utility* upper bound measures, i.e., *Probable Maximum Utility (PMU)* and *Neighborhood Sub-tree Utility (NSU)*, to identify itemsets (or items) whose supersets may generate SHUIs. Both *PMU* and *NSU* measures internally utilize the information on the neighborhood of items (or distance between the items) to identify itemsets whose supersets may generate SHUIs. We will now present each of these two measures.

**4.1.1 Probable Maximum Utility.** *PMU* aims to prune items whose supersets in a database (or projected database) cannot be SHUIs. The *PMU* of an itemset in a database is defined in Definition 12, and is based on Definitions 7, 8, 9, 10, and 11.

**Definition 7. (Probable Maximum Utility of an item in a transaction).** Let  $N_{ij}$  denote the neighbors of an item  $ij \in I$ , i.e.,  $\exists i_k \in N_{ij}; dist^1(ij; i_k) \leq maxDist$ . The *probable maximum utility (PMU)* of an item in a transaction  $T_{ts}$ , denoted as  $pmu^1ij; T_{ts}^\circ$ , represents the sum of utilities of  $ij$  and its neighboring items in  $T_{ts}$ , i.e.,  $pmu^1ij; T_{ts}^\circ = u^1ij; T_{ts}^\circ + \sum_{i_k \in N_{ij}} u^1i_k; T_{ts}^\circ$ .

**Example 8.** Consider the item  $a$  in Table 1. The neighbors of  $a$ , i.e.,  $N_a = \{b, c, e, g\}$  (see Table 5). The *probable maximum utility* of  $a$  in  $T_1$  is the sum of utilities of  $a$  and its neighboring items in  $T_1$ , i.e.,

$pmu^1a; T_1^\circ = u^1a; T_1^\circ + u^1b; T_1^\circ + u^1c; T_1^\circ + u^1e; T_1^\circ + u^1g; T_1^\circ = 200 + 150 = 350$  m. Note that the utilities of the remaining items (i.e.,  $g$  and  $f$ ) in  $T_1$  are not used while calculating  $pmu^1a; T_1^\circ$  because these two items are not neighbors of  $a$ . The above definition of *PMU* captures the maximum utility

of  $a$  and its neighboring items in  $T_1$ . We now extend this definition by considering a set of transactions (or a database).

**Definition 8. (PMU of an item in a database).** Let  $g^1ij^\circ$  denote the set of all transactions containing an item  $ij$ . The *PMU* of an item in a database, denoted as  $pmu^1ij^\circ$ , represents the sum of *probable maximum utility* of  $ij$  in all transactions containing  $ij$ , i.e.,  $pmu^1ij^\circ = \sum_{T_{ts} \in g^1ij^\circ} pmu^1ij; T_{ts}^\circ$ .

**Example 9.** The transactions containing  $a$  in Table 1 are as follows:  $T_1, T_2$  and  $T_9$ ; therefore,  $g^1a^\circ = \{T_1, T_2, T_9\}$ . The *PMU* of  $a$  in  $T_1$  is  $pmu^1a; T_1^\circ = 350$  m. Similarly,  $pmu^1a; T_2^\circ = 350$  m and  $pmu^1a; T_9^\circ = 850$  m. Note that the *PMU* of  $a$  in the entire database is  $pmu^1a^\circ = pmu^1a; T_1^\circ + pmu^1a; T_2^\circ + pmu^1a; T_9^\circ = 350 + 350 + 850 = 1550$  m, i.e.,  $a$  with all its neighbors has resulted in a *utility* (or congestion length) of 1,550 m.

The above definition captures the *utility* upper bound of an item with respect to its neighbors. Property 1 defines a pruning technique based on Definition 8. The correctness of this pruning technique is shown in Theorem 4.1, which is based on Properties 2 and 3.

**Property 1. (Pruning items using PMU)** For an item  $ij \in I$ , if  $pmu^1ij^\circ < minUtil$ , then  $ij$  can be pruned from the database because neither  $ij$  nor its supersets will be SHUIs.

**Example 10.** The *pmu* values for the items  $a; b; c; d; e; f$ , and  $g$  in Table 1 are 1550, 2650, 2550, 3250, 2100, 1450 and 1350, respectively. If the user-specified  $minUtil = 1500$ , then the items " $f$ " and " $g$ " can be pruned from the database because  $pmu^1f^\circ < minUtil$  and  $pmu^1g^\circ < minUtil$ .

**Property 2.** Let  $\mathcal{D} \subseteq D$  be a projected database containing items  $f, ij \in N_{ij}, g$ . Let  $\mathcal{P} \subseteq I$  be a set of all items in  $\mathcal{D}$ . The *utility* of  $ij$  in  $\mathcal{D}$  will always be less than or equal to  $pmu^1ij^\circ$ , i.e.,  $u^1ij; \mathcal{D} \leq pmu^1ij^\circ$ .

**Property 3. (Utility upper bound of PMU.)** In the projected database  $\mathcal{D}$ ,  $u^1X^\circ \leq pmu^1ij^\circ$ , as  $i_k \in X$ .

**THEOREM 4.1.** For an item  $ij \in I$ , if  $pmu^1ij^\circ < minUtil$ , then neither  $ij$  nor its supersets can be SHUIs.

**PROOF.** In  $\mathcal{D}$ , if  $pmu^1ij^\circ < minUtil$ , then  $u^1ij; \mathcal{D} < minUtil$  (see Property 2); thus,  $ij$  cannot be a SHUI. Moreover, for every itemset  $X$  in  $\mathcal{D}$ ,  $u^1X^\circ < minUtil$  (see Property 3); thus, all supersets of  $ij$  will not generate any SHUI. Hence, the validity of the algorithm is proved.

The items that are not pruned by Property 1 are known as **secondary items**. Definition 9 defines the secondary items. The SHUI-Miner algorithm does a depth-first search on the itemset lattice of secondary items to identify all SHUIs in the data.

**Definition 9. (Secondary items.)** An item  $ij \in I$  is a secondary item if  $pmu^1ij^\circ \geq minUtil$ .

**Example 11.** Continuing with the previous example, the secondary items in Table 1 are  $a; b; c; d$ , and  $e$ .

We now generalize Definition 8 by considering the notion of itemset. This generalization facilitates pushing the above pruning technique to the lower levels of the itemset lattice.

**Definition 10. (Neighboring items that can extend an itemset.)** Let  $SI$  denote the set of secondary items. Let  $\sigma$  denote an order of secondary items in  $PMU$  in an ascending order. Let  $\alpha$  be an itemset. Let  $E^1\alpha^o$  denote the set of all secondary items that can be used to extend  $\alpha$  according to the depth-first search, i.e.,  $E^1\alpha^o = \{z \mid z \subseteq SI \wedge z \not\subseteq \alpha; \exists x \subseteq \alpha, z \setminus x \subseteq SI \setminus \alpha\}$ . Let  $N^1\alpha^o = \bigcup_{i \in \sigma} N_{i_k}^o \setminus E^1\alpha^o$  denote the set of all neighboring items that can be used to extend  $\alpha$  according to the depth-first search to identify SHUIs.

**Example 12.** The  $PMU$  in ascending order of secondary items in Table 1 is  $SI = \{a; e; c; b; d\}$ . Let  $\alpha = a$  be an itemset. The  $E^1\alpha^o = \{e; c; b; d\}$ . The Neighborhood of  $a$  is  $N^1\alpha^o = \{e; c; b; d\}$  (see Table 5). Let  $\alpha = ae$ , then  $E^1\alpha^o = \{c; b; d\}$ . The Neighboring items that can extend  $ae$ , i.e.,  $N^1\alpha^o = N^1a^o \setminus N^1e^o \setminus E^1\alpha^o = \{e; c; b; d\} \setminus \{e; c; b; d\} \setminus \{c; b; d\} = \{c; b; d\}$ , i.e., the itemset  $ae$  will be extended by only considering the items  $c$  and  $b$ . The item  $d$  will not be utilized for extending  $ae$  because  $d$  is not a neighbor of  $a$ .

**Definition 11. (PMU of an itemset in a transaction)** The  $PMU$  of an itemset  $\alpha$  in a transaction  $T_{ts}$  is defined as  $PMU^1\alpha; T_{ts}^o = u^1\alpha; T_{ts}^o + \sum_{i \in \sigma} T_{ts}^o \wedge i \in N^1\alpha^o \wedge i \subseteq \alpha \wedge u^1i; T_{ts}^o$ .

**Example 13.** In Table 1,  $ae$  has appeared in the transaction  $T_9$ , which in  $PMU$  in an ascending order of items is  $\{1^o; 1^e; 3^o; 1^c; 2^o; 1^b; 2^o; 1^g; 1^o\}$ . The  $PMU$  of  $ae$  in  $T_9$  is the sum of utilities of  $ae$  and the neighboring items that are appearing in the sorted  $T_9$  after  $e$ , i.e.,  $pmu^1ae; T_9^o = u^1ae; T_9^o + u^1c; T_9^o + u^1b; T_9^o = 550 + 100 + 200 = 850$  m.

**Definition 12. (PMU of an itemset in a database)** The  $PMU$  of an itemset  $\alpha$  in a database is  $PMU^1\alpha^o = \sum_{T_{ts} \in \mathcal{T}} pmu^1\alpha; T_{ts}^o$ .

**Example 14.** In Table 1,  $ae$  has appeared only in the transaction  $T_9$ ; therefore,  $g^1\alpha^o = \{T_9\}$ . The  $pmu$  of  $ae$  in the entire database, i.e.,  $pmu^1ae^o = \sum_{T_9} pmu^1ae; T_9^o = 850$ , i.e., the maximum utility any superset of  $ae$  (containing only the neighboring items of  $a$  and  $e$ ) can have in Table 1 is 850 m.

The above definition represents the maximum utility achievable by any superset of an itemset (or an item) in the data. Thus, if the  $pmu$  of an itemset fails to satisfy  $minUtil$ , then we stop exploring its supersets. The accuracy of this algorithm is based on Property 4 as shown in Theorem 4.2. This theorem generalizes Property 1.

**Property 4. (Overestimation using the  $PMU$  of an itemset)** Let  $\alpha$  be an itemset and  $z$  be an item in  $N^1\alpha^o$ . Let  $\beta$  be an extension of  $\alpha$  such that  $z \subseteq \beta$ . The relationship  $PMU^1\alpha \lceil z^o \geq u^1\beta^o$  holds (in the projected database of  $\alpha$ ), i.e.,  $PMU$  is an overestimating function.

**THEOREM 4.2. (Pruning an item in all sub-trees using the  $PMU$ ).** Let  $\alpha$  be an itemset and  $z \subseteq N^1\alpha^o$ . If  $PMU^1\alpha; z^o < minUtil$ , then all extensions of  $\alpha$  containing  $z$  have low utility, i.e., item  $z$  can be ignored while exploring all sub-trees of  $\alpha$ .

**PROOF.** The correctness is straightforward to prove from Property 4.

**4.1.2 Neighborhood sub-tree utility.** The  $PMU$  measure identifies secondary items (or itemsets) whose supersets may be SHUIs in the database. We use a variant of *sub-tree utility* [27], called *Neighborhood sub-tree utility*  $^1NSU^o$ , to identify these secondary items whose projections (or depth-first search in the itemset lattice) will result in identifying all SHUIs. The  $NSU$  is defined in Definition 13 and illustrated in Example 15. The secondary items whose projections will result in identifying SHUIs are referred to as *primary items*. The primary items are defined in Definition 14 and illustrated in Example 16. The relationship between  $PMU$  and  $NSU$  is given in Property 5.

**Definition 13. (Neighborhood sub-tree utility)** Let  $\alpha$  be an itemset and  $z \subseteq N^1\alpha^o$  be an item that can extend  $\alpha$  according to the depth-first search in the itemset lattice (or set enumeration tree). The neighborhood sub-tree utility of  $z$  with respect to  $\alpha$ , denoted as  $NSU^1\alpha; z^o = \sum_{T_{ts} \in \mathcal{T}} [z^o \supseteq u^1\alpha; T_{ts}^o + u^1z; T_{ts}^o + \sum_{i \in \sigma} T_{ts}^o \wedge i \in N^1\alpha^o \wedge i \subseteq z^o \wedge u^1i; T_{ts}^o]$ .

**Example 15.** The  $PMU$  in an ascending order of secondary items in Table 1 is  $a; e; c; b$ , and  $d$ . Let us consider item  $d$ , which has  $pmu$  value of 3,250. Let  $\alpha = a$ . The neighborhood sub-tree utility of  $\alpha \lceil d$ ,  $NSU^1\alpha; d^o = 300\% + 300\% + 100\% + 200\% = 600$ . Because  $pmu^1d^o < minUtil$  and  $NSU^1d^o < minUtil$ , it turns out that  $d$  will generate SHUIs in the projections of another items. The projection (or depth-first search) on  $d$  will not result in any SHUI.

**Property 5.** For an itemset  $X$ ,  $PMU^1X^o \geq NSU^1X^o$ .

**Definition 14. (Primary items).** Given the  $PMU$  in an ascending order of secondary items, a secondary item  $ij \subseteq SI$  is said to be a primary item if  $NSU^1ij^o < minUtil$ .

**Example 16.** Consider the secondary items  $a$  and  $d$ , which have minimum and maximum  $pmu$  value in Table 1. The  $NSU$  values for these two secondary items, i.e.,  $NSU^1a^o = 1550$  and  $NSU^1d^o = 600$ . Because  $NSU^1a^o < minUtil$ , we consider  $a$  as a primary item. We do not consider  $d$  as a primary item because  $NSU^1d^o < minUtil$ .

**4.1.3 Comparison of  $PMU$  and  $NSU$  measures against existing pruning measures.** In the literature, researchers have employed *Transaction Weighted Utility* ( $TWU$ ) [15], *local utility* ( $LU$ ), and *sub-tree utility* ( $STU$ ) [27] as utility upper bound measures to reduce the computational cost of identifying SHUIs in (non-binary) transactional databases. These measures are defined in Definitions 15, 16, and 17. The proposed  $PMU$  measure is a tighter utility upper bound compared to  $TWU$  and  $LU$  measures (see Theorem 4.3). Similarly, the proposed  $NSU$  measure is a tighter utility upper bound compared to  $STU$  measure (see Theorem 4.4), i.e., the proposed  $PMU$  and  $NSU$  measures can help in determining the SHUIs by generating fewer secondary items and primary items compared to the existing HUIM algorithms [9, 27].

In the next subsection, we present SHUI-Miner algorithm using primary and secondary items.

**Definition 15. (Transaction Weighted Utility of an itemset.)** In a database  $D$ , the  $TWU$  of an itemset  $\alpha$ , denoted as  $TWU^1\alpha^o$ ,

represents the sum of utilities of all transactions containing  $\alpha$ , i.e.,  $TWU^1\alpha^0 = \sum_{T_{ts} \in \mathcal{T}} \sum_{i_j \in T_{ts}} u^1 i_j; T_{ts}^0$ , where  $g^1\alpha^0$  represents the set of transactions containing  $\alpha$ .

**Definition 16. (Local utility of an itemset.)** In a database  $D$ , the  $LU$  of an itemset  $\alpha$  [  $z$ , denoted as  $LU^1\alpha^0 = \sum_{T_{ts} \in \mathcal{T}} [z^0 u^1\alpha; T_{ts}^0 + \sum_{i_j \in T_{ts} \wedge i_j \in E^1} u^1 i_j; T_{ts}^0$ .

**Definition 17. (Sub-tree utility)** Let  $\alpha$  be an itemset and  $z$  be an item that can extend  $\alpha$  according to the depth-first search in the itemset lattice (or set enumeration tree). The sub-tree utility of  $z$  with respect to  $\alpha$ , denoted as  $STU^1\alpha; z^0 = \sum_{T_{ts} \in \mathcal{T}} [z^0 u^1\alpha; T_{ts}^0 + u^1 z; T_{ts}^0 + \sum_{i_j \in T_{ts} \wedge i_j \in E^1} [z^0 u^1 i_j; T_{ts}^0$ .

**Property 6.** For an itemset  $\alpha$ ,  $TWU^1\alpha^0 = LU^1\alpha^0$  [27].

**THEOREM 4.3.** For an itemset  $\alpha$ ,  $TWU^1\alpha^0 = LU^1\alpha^0 = PMU^1\alpha^0$ .

**PROOF.** For an itemset  $\alpha$ ,  $N^1\alpha^0 = E^1\alpha^0$  (see Definition 10); thus,  $LU^1\alpha^0 = PMU^1\alpha^0$ . From Property 6, it turns out that  $TWU^1\alpha^0 = PMU^1\alpha^0$ ; thus,  $TWU^1\alpha^0 = LU^1\alpha^0 = PMU^1\alpha^0$ . Hence, the validity of the algorithm is proved.

**THEOREM 4.4.** For an itemset  $\alpha$  [  $z$ ,  $STU^1\alpha; z^0 = NSU^1\alpha; z^0$ .

**PROOF.** Because  $N^1\alpha$  [  $Z^0 = E^1\alpha$  [  $z^0$ , it turns out that  $\sum_{T_{ts} \in \mathcal{T}} [z^0 u^1\alpha; T_{ts}^0 + u^1 z; T_{ts}^0 + \sum_{i_j \in T_{ts} \wedge i_j \in N^1} [z^0 u^1 i_j; T_{ts}^0 = \sum_{T_{ts} \in \mathcal{T}} [z^0 u^1\alpha; T_{ts}^0 + u^1 z; T_{ts}^0 + \sum_{i_j \in T_{ts} \wedge i_j \in E^1} [z^0 u^1 i_j; T_{ts}^0$ , i.e.,  $NSU^1\alpha; z^0 = STU^1\alpha; z^0$ . Hence, this confirms the validity of the algorithm.

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### Algorithm 1 SHUI

- 1: **input:**  $D$  is a temporal database,  $SD$  is a spatial database,  $UD$  is an external utility database,  $minUtil$  is a user-specified minimum utility constraint, and  $maxDist$  is a user-specified maximum distance constraint
- 2: **output:** A set of SHUIs
- 3:  $N = Neighbors^1SD; maxDist^0$ . Let  $N^1 i_j^0$  denote the set of all neighboring items for  $i_j \in I$ .
- 4: Let  $\alpha$  denote an itemset that needs to be extended. Initially, set  $\alpha = \emptyset$ ;
- 5: Scan the temporal database to determine the  $PMU$  for every item  $i_j \in I$ .
- 6:  $Secondary^1\alpha^0 = \{i_j | i_j \in I \wedge PMU^1 i_j^0 > minUtil\}$
- 7: Let  $\beta$  be the total order of  $PMU$  in an ascending order of their values in  $Secondary^1\alpha^0$ ;
- 8: Scan  $D$  to remove each item  $i < Secondary^1\alpha^0$  from the transactions, sort items in each transaction according to  $\beta$ , and delete empty transactions;
- 9: Calculate *neighborhood sub-tree utility* for all items in  $secondary^1\alpha^0$  by scanning the database  $D$  once using utility-bin array;
- 10:  $Primary^1\alpha^0 = \{z \in secondary^1\alpha^0 | NSU^1\alpha; z^0 > minUtil\}$ ;
- 11:  $RecursiveSearch^1\alpha; D; Primary^1\alpha^0; Secondary^1\alpha^0; minUtil^0$ ;

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### Algorithm 2 Neighbors

- 1: **input:** SD:spatial database;  $maxDist$ : maximum distance
- 2: **output:**  $N < item; neighbors >$ : A two dimensional hashmap with records of items and their neighbors
- 3: **for** each item  $i_j \in SD$  **do**
- 4:   **for** each item  $i_k \in SD$  **do**
- 5:     **if**  $(i_j, i_k)$  and  $(Dist^1 i_j; i_k^0 > maxDist)$  **then**
- 6:       Add  $i_k$  as a neighbor of  $i_j$  in  $N$ .
- 7:     **end if**
- 8:   **end for**
- 9: **end for**

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### Algorithm 3 RecursiveSearch

- 1: **input :**  $\alpha$  is an itemset,  $\beta \subseteq D$  is the  $\alpha$  projected database,  $Primary^1\alpha^0$  is the primary items of  $\alpha$ ,  $Secondary^1\alpha^0$  is the secondary items of  $\alpha$ , and  $minUtil$  is minimum utility
- 2: **output:** the set of all SHUIs that are extensions of  $\alpha$
- 3: **for** each item  $i \in Primary^1\alpha^0$  **do**
- 4:    $\beta = \alpha \cup \{i\}$ ;
- 5:   Scan  $\alpha$ - $D$  to calculate  $u^1\beta^0$ . Create a projected database,  $\beta \subseteq D$ .
- 6:   **if**  $u^1\beta^0 > minutil$  **then**
- 7:     output  $\beta$  as a spatial high utility itemset
- 8:   **end if**
- 9:   Calculate  $stu^1\beta; z^0$  and  $lu^1\beta; z^0$  for all item  $z \in Secondary^1\alpha^0$  by scanning  $\beta$ - $D$  once, using two utility-bin array;
- 10:    $Primary^1\beta^0 = \{z \in Secondary^1\alpha^0 | NSU^1\beta; z^0 > minutil \text{ and } z \in N\}$ ;
- 11:    $Secondary^1\beta^0 = \{z \in Secondary^1\alpha^0 | PMU^1\beta; z^0 > minutil \text{ and } z \in N\}$ ;
- 12:   RecursiveSearch( $\beta, \beta$ - $D, Primary^1\beta^0, Secondary^1\alpha^0, minutil, minSup$ );
- 13: **end for**

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## 4.2 Proposed algorithm

The SHUI-Miner is presented in Algorithms 1, 2, and 3. Because the calculation of  $PMU$  and  $NSU$  for a projected database of  $\alpha$  is straightforward, we have not presented the related algorithms in this study. The terms "projected transaction" and "projected database" are defined in Definitions 18 and 19, respectively. We now illustrate these three algorithms using the databases shown in Tables 1, 2, and 3. Let  $minUtil = 1500$  and  $maxDist = 5$ .

**Definition 18. (Projected transaction.)** The projection of a transaction  $T \in D$  using an itemset  $\alpha$  is denoted as  $\alpha \cdot T$  and defined as  $\alpha \cdot T = [g^1 T \wedge i_2 N^1 \cup i]$ .

**Example 17.** Let  $\alpha = a$ . In Table 1, the item  $a$  appears in  $T_1$ ; therefore, the projected transaction of  $a$ , i.e.,  $a \cdot T = \{1b; 3^0g\}$ . Other items in  $T_1$  are not included in  $a \cdot T$  because they are not neighbors of  $a$ .

**Definition 19. (Projected database.)** The projection of a database  $D$  using an itemset  $\alpha$  is denoted as  $\alpha \cdot D$  and defined as the multiset  $\alpha \cdot D = [g^1 T_2 \wedge i_2 T_1; f^1 \alpha \cdot T_g]$ .

ts	Items
1	<sup>1</sup> a; <sup>2</sup> b; <sup>3</sup> c
2	<sup>1</sup> a; <sup>1</sup> c; <sup>3</sup> c
3	<sup>1</sup> d; <sup>3</sup> c
4	<sup>1</sup> c; <sup>4</sup> b; <sup>3</sup> c; <sup>1</sup> d; <sup>1</sup> c
6	<sup>1</sup> e; <sup>1</sup> c; <sup>4</sup> b; <sup>4</sup> c; <sup>1</sup> d; <sup>1</sup> c
9	<sup>1</sup> a; <sup>1</sup> e; <sup>3</sup> c; <sup>1</sup> c; <sup>2</sup> b; <sup>2</sup> c

**Table 6: Sorted temporal database generated from Table 1**

**Example 18.** In Table 1, item  $a$  appears in  $T_1, T_2$ , and  $T_9$ ; therefore,  $a \ D = ff^1b; 3^0g; f^1c; 3^0g; f^1e; 3^0; 1^c; 2^0; 1^b; 2^0gg$ .

To identify neighbors for each item in the data (Line 3 in Algorithm 1 and entire Algorithm 2), the spatial database shown in Table 3 is used. The list of items and their corresponding neighbors are shown in Table 5. Next, we set  $\alpha = ;$  and calculate  $PMU$  values for each item in Table 1. The list of  $PMU$  values is  $ffa : 1550g; fb : 2650g; fc : 2550g; fd : 3250g; fe : 2100g; ff : 1450g; fg : 1350gg$  (lines 4 and 5 in Algorithm 1). Next, secondary items with respect to  $\alpha$  is generated by comparing  $PMU$  of each item with  $minUtil$  (line 6 in Algorithm 1). The generated secondary items are sorted in  $PMU$  in an ascending order ; thus,  $Secondary^1\alpha^0 = fa; e; c; b; dg$  (line 7 in Algorithm 1). Next, we prune all non-secondary items in  $D$ , sort the remaining items in each transaction in order, and delete empty transactions. The resulting sorted temporal database containing only secondary items is shown in Table 6 (line 8 in Algorithm 1). Next, we calculate  $NSU$  for each item in  $Secondary^1\alpha^0$ . The  $NSU$  values for the ordered secondary items is  $ffa : 1550g; fe : 2000g; fc : 3300g; fb : 1850g; fd : 600gg$  (line 9 in Algorithm 1). The items that have  $NSU$  value greater than  $minUtil$  are considered as  $primary$  items, i.e.,  $Primary^1\alpha^0 = fa; e; c; bg$  (line 10 in Algorithm 1). Next, we call RecursiveSearch function to generate all SHUIs from the database (line 11 in Algorithm 1).

The RecursiveSearch (i.e., Algorithm 3) works as follows. First, start with the first item  $a$  in the  $Primary^1\alpha^0$ ; moreover, because  $\alpha = ;$ ,  $\beta = a$  and we scan  $D$  (i.e.,  $\alpha \ D$ ) and calculate  $u^1\beta^0 = u^1\alpha^0 = 350$ . Next, we construct a projected database  $\beta \ D$  (i.e.,  $a \ D$ ) with only neighboring items of  $a$ . Figure 5 (a) shows the projected database of  $a$  generated from Table 6. As  $u^1\alpha^0 = minUtil$ ,  $a$  will not be generated as a SHUI. Next, for each item  $z$  in  $\beta \ D$ , we calculate  $PMU$  and  $NSU$  values. Figure 5(b) shows the  $PMU$  and  $NSU$  values calculated for all items in  $a \ D$ . The items having  $NSU$  greater than or equal to  $minUtil$  are considered as  $primary$  items (i.e.,  $primary^1\beta^0$ ). As all items in  $a \ D$  have  $NSU$  value less than  $minUtil$ , we stop creating the projected databases for the primary item  $a$  and move to the next primary item  $e \ 2 \ Primary^1\alpha^0$ . The projected database of  $e$ , i.e.,  $e \ D$ , is shown in Figure 5(c). Next, we calculate  $PMU$  and  $NSU$  values for all items in  $e \ D$ . The items  $c$  and  $b$  in  $e \ D$  have  $NSU \geq minUtil$ ; thus, we consider  $primary^1e^0 = fc; bg$ . Next, we construct a projected database for  $ec$ , i.e.,  $(ec \ D)$ , as shown in Figure 5(e). We then calculated  $PMU$  and  $NSU$  for all neighbouring items in  $ec \ D$ , as shown in Figure 5(f). It can be observed from Figure 5(f) that we have not calculated  $PMU$  and  $NSU$  for the item  $b$  in  $ec \ D$  because  $b$  is not a neighbor of  $c$ . The above process is repeated for all primary items. All SHUIs

ts	Items
1	(b, 3)
2	(c, 3)
9	(e, 3), (c, 2), (b, 2)

(a)

Items	PMU	NSU
e	850	850
c	1200	750
b	900	650

(b)

ts	Items
6	(c, 4), (b, 4), (d, 1)
9	(c, 2), (b, 2)

(c)

Items	PMU	NSU
c	2000	2000
b	2000	1500
d	1250	250

(d)

ts	Items
6	(b, 4), (d, 1)
9	(b, 2)

(e)

Items	PMU	NSU
b	-	-
d	1250	650

(f)

**Figure 5: Recursive mining of SHUI-Miner. (a) Projected database of  $a$  (i.e.,  $a \ D$ ), (b)  $PMU$  and  $NSU$  values of the items in  $a \ D$ , (c) projected database of  $e$  (i.e.,  $e \ D$ ), (d)  $PMU$  and  $NSU$  values of the items in  $e \ D$ , (e) projected database of  $ec$  (i.e.,  $ec \ D$ ), and (f)  $PMU$  and  $NSU$  values of the items in  $ec \ D$**

generated by SHUI-Miner for the database shown in Table 1 are presented in Table 4.

## 5 EXPERIMENTAL RESULTS

Because there exists no algorithm for identifying SHUIs in spatiotemporal databases, we only evaluate the performance of SHUI-Miner. The SHUI-Miner algorithm was written in C++ and executed on a machine with 1.5 GHz processor and 4GB RAM. The experiments have been conducted on two real-world databases, namely, *congestion database* and *pollution database*. The XRAIN database was used to demonstrate the usefulness of SHUIs generated from congestion database. The open source application, QGIS, is used to display some of the interesting SHUIs generated from both databases. We now explain all three databases.

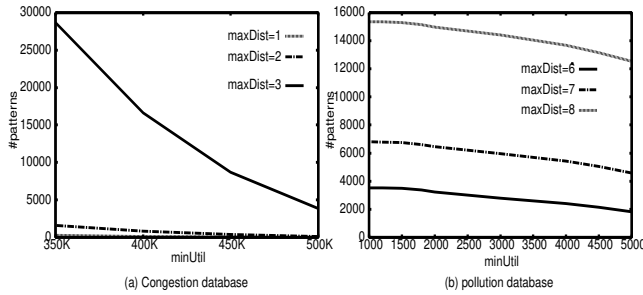
The (traffic) congestion database is provided by JARTIC. The attributes of the congestion database is shown in Table 7. Sample data of JARTIC is shown in Figure 2(a). The data from each sensor was collected over an interval of 5 min. The primary attributes for this database are as follows: *mesh code* and *Road ID*. The distance between the road segments was calculated using geodesic distance. In the experiment, we used road congestion data generated on July 17, 2015, for Kobe, Japan. On this day, Kobe was hit by Typhoon Nangka, which brought over 2 feet of rain and caused 550,000 people to leave Kobe. The source database contained 39,873 transactions and 2,412 items (or RIDs). To record the transactions, we used UTC time. Without loss of generality, original congestion database was split into temporal database, spatial database, and external utility database. The internal utility for the items in the temporal database was determined by counting the number of occurrences of Road ID (RID) at a particular timestamp.



Attribute	Type of attribute
Timestamp	interval data
Mesh code	categorical data
RID	categorical data
Location	line
Con. length	continuous data

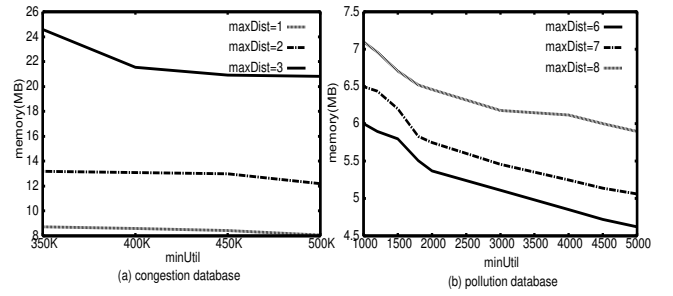
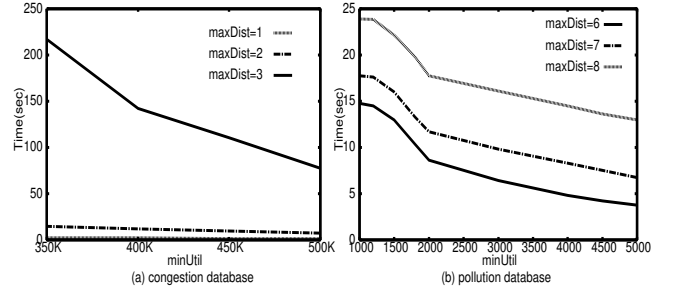
**Table 7: Attributes of congestion database**

Attribute	Type of attribute
Timestamp	interval data
Sensor ID	categorical data
Sensor location	point
PM <sub>2.5</sub>	continuous data

**Table 8: Attributes of pollution database**

**Figure 6: SHUIs generated in pollution database**

The air pollution, particularly PM<sub>2.5</sub> (or simply, PM<sub>2.5</sub>) concentrate, is the major cause for many cardio-respiratory problems reported in Japan. In particular, the health of elderly people was severely affected by PM<sub>2.5</sub>. To control and monitor air pollution, AEROS was setup by the Ministry of Environment, Japan. The data generated by various sensors in AEROS helped develop the air pollution database [20]. In this study, we focused on PM<sub>2.5</sub>, and similar studies were performed for other pollutant concentrates such as NO<sub>2</sub> and O<sub>3</sub>. The unit for measuring PM<sub>2.5</sub> is microgram per cubicmeter ( $\mu\text{g}\cdot\text{m}^3$ ). The attributes of pollution database are presented in Table 8. The sample data of this table is shown in Figure 4(a). The air pollution data was generated at an interval of 1 h for 24 h a day. For our experiments, we used air pollution data of the following three days: April 21, 2018; April 22, 2018; and April 23, 2018. The pollution database has 97,651 transactions and 1,026 items (or sensor ids). To record the transactions, we used UTC time. Without loss of generality, the pollution database was split into temporal, spatial, and external utility databases. Since each sensor produces only one value in an hour (i.e., transaction), the internal utility for the items in temporal database is set as 1.

XRAIN data is a high-resolution precipitation raster data provided by Japan Meteorological Agency (JMA). In this study, we used the precipitation data of Typhoon Nangka, which struck Japan on July, 17, 2015.


**Figure 7: Memory consumed by SHUI-Miner**

**Figure 8: Runtime requirements of SHUI-Miner**

Figures 6(a) and (b) show the number of SHUIs generated in congestion and pollution databases at different *minUtil* and *maxDist* values, respectively. The following observations can be drawn from these two figures. <sup>1</sup>*i*<sup>o</sup> Increase in *maxDist* results in increase in SHUIs because increase in *maxDist* increases the neighborhood size of the items, which thereby enables items to combine with more items and generate SHUIs and *(ii)* Increase in *minUtil* results in decrease in SHUIs. This is because many itemsets fail to satisfy the increased *minUtil* value.

Figures 7(a) and (b) show the memory requirements of SHUI-Miner in congestion and pollution databases at different *minUtil* and *maxDist* values, respectively. The following observations can be drawn from these two figures. <sup>1</sup>*i*<sup>o</sup> The memory requirements of SHUI-Miner increases with an increase in *maxDist* because increase in SHUIs results in an increase in *maxDist*. *(ii)* The memory requirements of SHUI-Miner decreases with increase in *minUtil* because fewer SHUIs get generated with increase in *minUtil*. *(iii)* Moreover, it is observed that SHUI-Miner takes less time to identify SHUIs in congestion and pollution databases at large or small threshold values.

Figures 8(a) and (b) show the runtime requirements of SHUI-Miner algorithm in congestion and pollution databases at different *minUtil* and *maxDist* values. The following observations can be drawn from these two figures. <sup>1</sup>*i*<sup>o</sup> The runtime of SHUI-Miner increases with an increase in *maxDist* because of the increase in the number of SHUIs. *(ii)* The runtime of SHUI-Miner decreases with the increase in *minUtil* because of the decrease in SHUIs as a result of an increase in *minUtil*. *(iii)* Moreover, it can be observed that SHUI-Miner is efficient for large or small threshold values.

S.No.	SHUI	Utility	Area
1	f807; 799; 792; 790; 813; 812g	13959	Tokyo
2	f175; 177; ; 176; 174; 173g	8616	Hiroshima
3	f516; 520; 517; 519g	8556	Nagoya
4	f352; 354; 356; 343g	7822	Osaka

**Table 9: Some of the interesting SHUIs generated from pollution database**

## 5.1 Case studies

**5.1.1 Improving traffic safety.** To demonstrate the usefulness of SHUIs in traffic congestion data, we have divided the congestion data of July, 17, 2015, into hourly intervals and applied SHUI-Miner algorithm to each hourly congestion data to identify SHUIs (i.e., sets of road segments where a lot of congestion was observed within the traffic network). The  $maxDist = 1$  km and  $minUtil = 10000$  m (=10 km), i.e., a total of at least 10 km of congestion should be observed in a set of neighboring roads in an hour. Figure 9 shown hourly congestion reported from 6:00 hours to 14:00 hours. The black lines in each of these figures represent the road segments where congestion was observed by a sensor in the entire one hour interval of time. On these black lines, we have overlaid colored lines that represent road segments (or SHUIs) generated in hourly congestion data. Road segments with same color represent a SHUI. Moreover, hourly XRAIN data of the typhoon Nangka was overlaid on the road segments. The following observations can be drawn from these figures: (i) roads with heavy congestion (i.e., SHUIs) vary with time (ii) roads with heavy congestion were observed in areas with high precipitation (Figures 9(d)-(f)). Thus, the knowledge of SHUIs (or heavy congested roads) can be very useful to the traffic control room in diverting the traffic.

Some of the authors of this paper have previously developed a model to predict congestion on roads using XRAIN data [22]. Applying SHUIM algorithm on this predicted congestion database provides prior information regarding the road segments (SHUIs) where heavy congestion might happen. The users in traffic control room can utilize this knowledge for various purposes such as diverting the traffic, alerting the pedestrians to execute caution while crossing the roads, and suggesting patrol paths for the police.

**5.1.2 Identifying highly air polluted locations.** Table 9 shows some of the SHUIs generated from air pollution database at  $maxDist = 7$  kilometers and  $minUtil = 5000 \mu g \cdot m^3$ . The spatial location of these sensors is shown in Figure 10. The exact location of the sensors presented in each SHUI is shown in Figures 11-14. It can be observed in these figures that large quantities of PM2.5 were observed at the sea ports (situated to the east coast of Japan). This information can be found very useful for devising policies for pollution control.

## 6 CONCLUSIONS AND FUTURE WORK

In this study, we exploited the spatiotemporal characteristics of the items within the data and proposed a flexible model for evaluating SHUI that exists in a spatiotemporal database. Two novel utility upper bound measures, *PMU* and *NSU*, have been introduced to reduce the search space of SHUIM. Moreover, we have theoretically shown that the proposed measures are tighter than the current utility upper bound measures when it comes to mining SHUIs. A

single pass algorithm has been proposed to effectively determine all SHUIs in spatiotemporal databases. Note that experimental results demonstrate that the proposed algorithm is efficient, and we also presented two case studies to demonstrate the effectiveness of SHUIs in real-world applications.

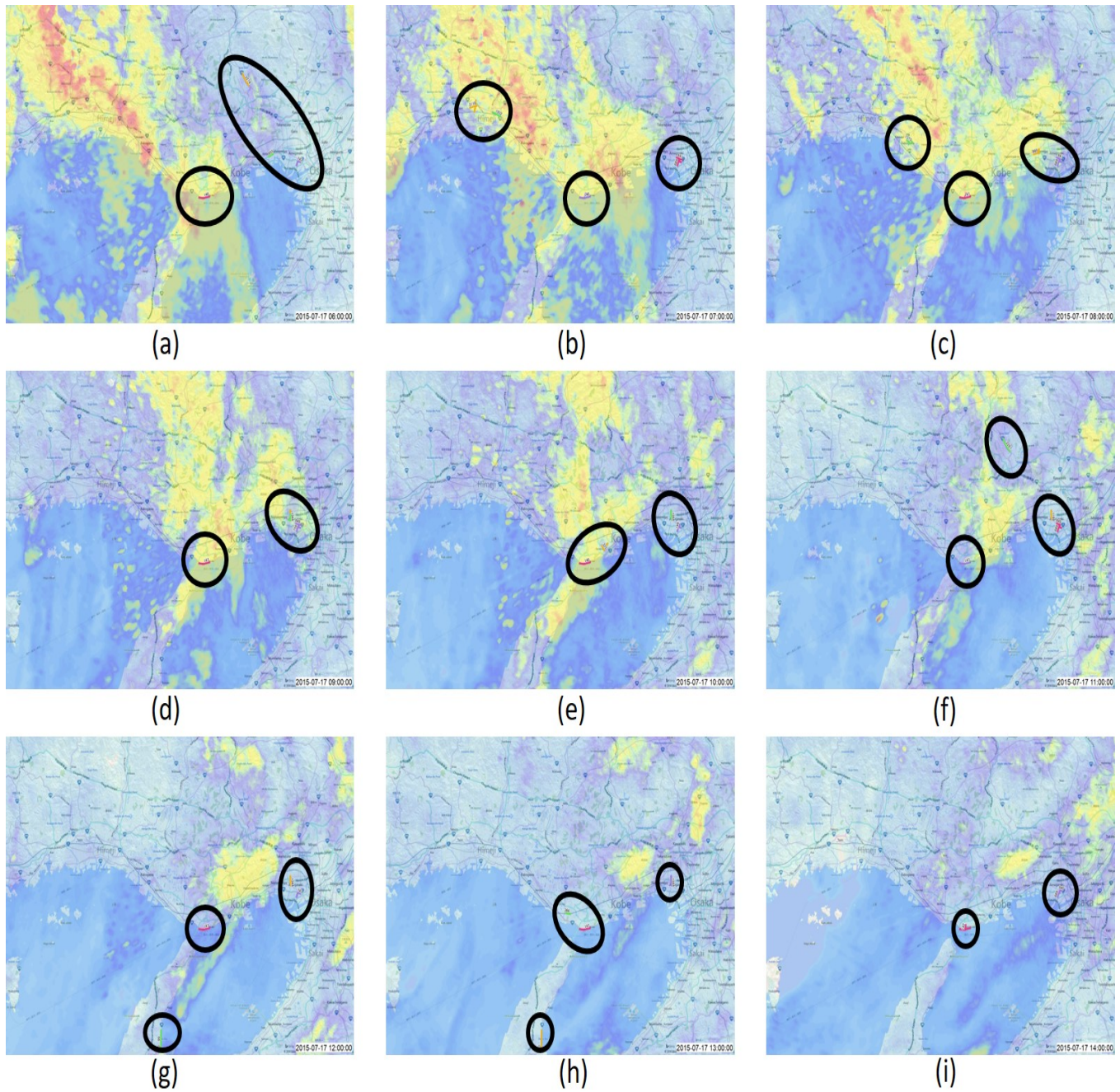
In this study, we examined SHUIM by considering only positive external utility values. As a part of our future work, we would like to extend the proposed model of SHUI by considering both positive and negative external utility values. We focused on identifying SHUIs in static spatiotemporal data, and it would be interesting to investigate SHUIM in graphs, data streams, and symbolic databases in future.

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**Figure 9: SHUIs generated from the hourly congestion data. XRain (or precipitation) data is overlaid at hourly intervals. High congested road segments have been shown within the dark ovals. (a) to (i) show some of the SHUIs generated from the hourly congestion data starting from 6 to 14 h (UTC time)**

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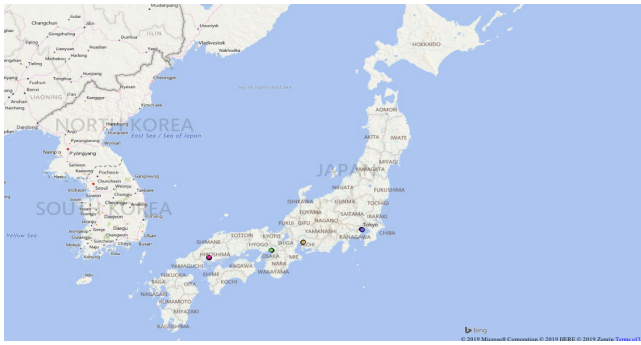


Figure 10: Spatial location of SHUIs generated in pollution data

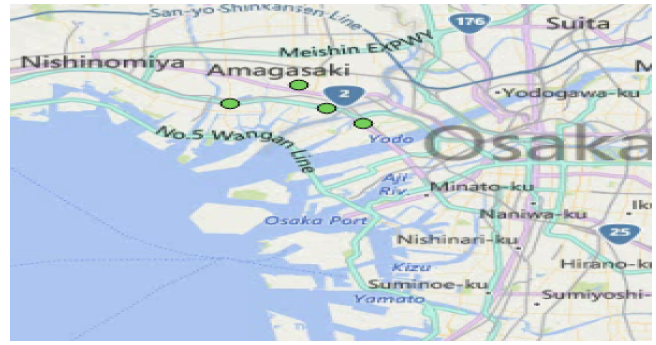


Figure 13: Sensors that recorded high quantities of PM2.5 in Osaka area

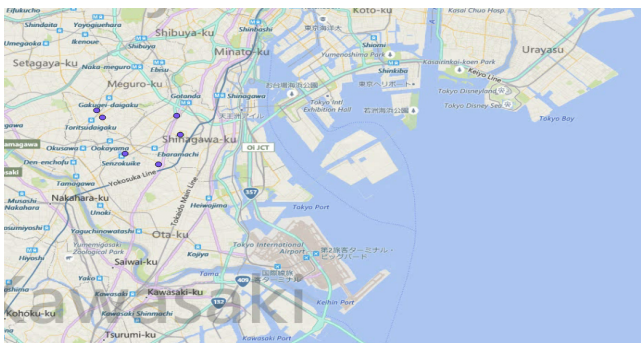


Figure 11: Sensors that recorded high quantities of PM2.5 in Tokyo area

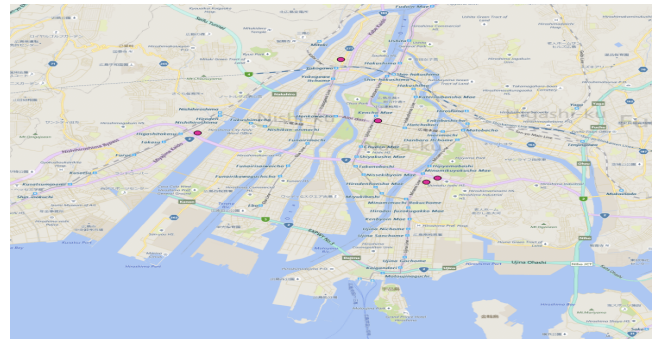


Figure 14: Sensors that recorded high quantities of PM2.5 in Hiroshima area

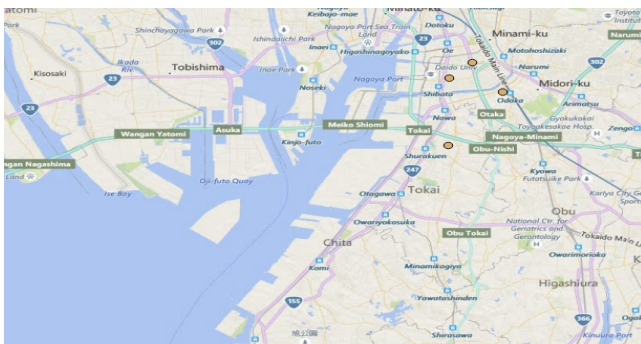


Figure 12: Sensors that recorded high quantities of PM2.5 in Nagoya area

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